National Open School of Trinidad and Tobago

LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT Five:
Simultaneous Equations and Inequalities

Lesson 10

Solving for Two Unknowns

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1 and 2

(Some helpful exercises and page numbers are given throughout the lesson, e.g. Ex 6o page 264)

INTRODUCTION

The equations we solved previously involved solving for one unknown. However, solving many real life problems involve solving for two unknowns. This lesson will introduce the skills and concepts needed to solve such problems.

OBJECTIVES

At the end of this lesson you will be able to:

a) Solve simultaneous linear equations in two unknowns algebraically;
b) Use simultaneous equations to solve word problems;
c) Solve a pair of equations in two variables when one equation is quadratic or non-linear and the other linear.

5.8 Simultaneous Equations

The Method of Elimination

With the elimination method make the magnitude of one of the unknowns equal. Once they are equal they can be eliminated. If the sign of the numbers in front of the equal unknowns are the same then subtract the two equations to eliminate them. If it is different add the two equations to eliminate them.

Example: Solve the following pairs of simultaneous equation (Ex 6o page 264):

$5x + 2y = 29$

$x - y = -4$

Solution:

We can make the y terms equal by multiplying the second equation by 2. The signs in front of the y terms are different therefore add the two equations after multiplying by 2 to eliminate the y terms.

$$\begin{array}{rl} & 5x + 2y = 29 \\ + & 2x - 2y = -8 \\ \hline & 7x + 0 = 21 \end{array}$$

$$\frac{7x}{7} = \frac{21}{7}$$

$x = 3$

Substitute $x = 3$ into either of the equations above and solve for *y.*

$5(3) + 2y = 29$

$15 + 2y = 29$

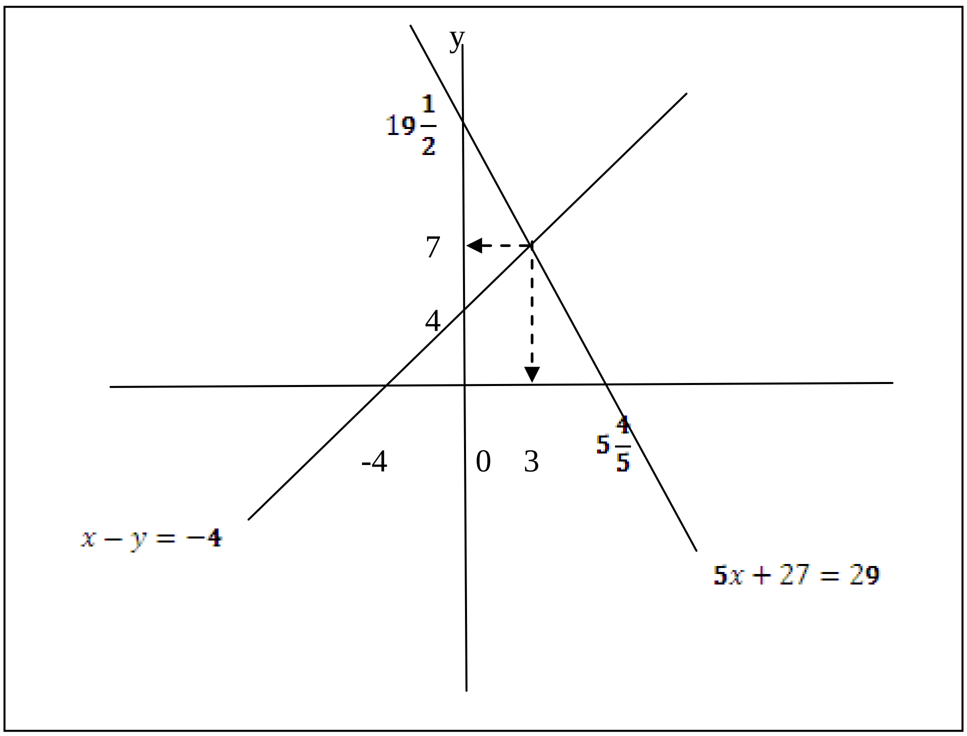
$15 - 15 + 2y = 29 - 15$

$2y = 14$

$\frac{2y}{2} = \frac{14}{2}$

$y = 7$

This solution can be shown graphically



The Method of Substitution

In this method make one of the unknowns the subject of one of the equations. Then substitute this in the other equation. When this is done we get one equation with one unknown that can be solved.

Example: Solve the following pairs of simultaneous equation (Ex 6o page 264):

$-2x - 3y = \frac{7}{2}$

$4x + y = -2$

Solution:

Make y the subject of the second equation.

$4x - 4x + y = -2 - 4x$

$y = -2 - 4x$

substitute $y = -2 - 4x$ in the first equation.

$-2x - 3(-2 - 4x) = \frac{7}{2}$

$-2x + 6 + 12x = \frac{7}{2}$

$6 + 10x = \frac{7}{2}$

$6 - 6 + 10x = \frac{7}{2} - 6$

$10x = -\frac{5}{2}$

$2 \times 10x = -\frac{5}{2} \times 2$

$20x = -5$

$$\frac{20x}{20} = \frac{-5}{20}$$

$$x = -\frac{1}{4}$$

Substitute $x = -\frac{1}{4}$ into $y = -2 - 4x$.

$$y = -2 - 4\left(-\frac{1}{4}\right)$$

$$y = -2 + 1$$

$$y = -1$$

ACTIVITY 1

Use both methods (elimination and substitution) to solve the following equations:

$$-5x + 2y = 24$$

$$-7x + 3y = 35$$

Word Problems

Some word problems can be solved using simultaneous equations. This involves translating the problem into two equations with two unknowns to be solved.

Example (Ex 6r page 274):

Ria bought 3 hot dogs and 5 hamburgers for \$32.75. If, however, Ria had bought 4 hot dogs and 4 hamburgers she would have paid \$29.00. Calculate the price Ria paid per hot dog and per hamburger, to the nearest cent.

Solution:

Let the cost of a hot dog be x and the cost of a hamburger be y. The two equations are therefore:

$$3x + 5y = \$32.75$$

$$4x + 4y = \$29.00$$

Using the elimination method we can eliminate the x terms by multiplying the first equation by 4 and the second equation by 3 and then subtracting the two equations.

$$\begin{array}{rl} & 12x + 20y = \$131.00 \\ - & 12x + 12y = \$87.00 \\ \hline & 0 + 8y = \$44.00 \end{array}$$

$$\frac{8y}{8} = \frac{\$44.00}{8}$$

$$y = \frac{\$44.00}{8}$$

$$y = \$5.50 .$$

Substitute $y = \$5.50$ into the first equation, $3x + 5y = \$32.75$.

$$3x + 5\left(\$5.50\right) = \$32.75$$

$3x + \$27.50 = \$32.75$

$3x + \$27.50 - \$27.50 = \$32.75 - \$27.50$

$3x = \$5.25$

$\frac{3x}{3} = \frac{\$5.25}{3}$

$x = \$1.75$

The cost for a hot dog is $1.75 and a hamburger is $5.50.

Solving simultaneous equations where one of the equations is non-linear (not a straight line) and the other is linear (a straight line).

Example:

Solve the pair of simultaneous equations (Ex 13m page 795):

$y = 2x^2 + 1$

$y = 7x + 5$.

Solution:

Note that the two expressions are equal since they are both equal to y .

Therefore we have:

$2x^2 + 1 = 7x + 5$

Bring all the terms on the left side of the equation.

$2x^2 - 7x + 1 = 7x - 7x + 5$

$2x^2 - 7x + 1 = 5$

$2x^2 - 7x + 1 - 5 = 5 - 5$

$2x^2 - 7x - 4 = 0$

This is a quadratic that can be solved by factorization.

Do you recall how to split the middle term? You may review how this is done from lesson 8.

$2 \times (-4) = -8$

$-8 \times 1 = -8$

$-8x + x = -7x$

Therefore we can use $-8x + x$ to split the middle term.

$2x^2 - 8x + x - 4 = 0$

Now factorize by grouping.

$(2x^2 - 8x) + (x - 4) = 0$

$2x(x - 4) + 1(x - 4) = 0$

$(x - 4)(2x + 1) = 0$

Put each bracket equal to 0 and solve for x .

$(x - 4) = 0$

$x - 4 + 4 = 0 + 4$

$x = 4$

and

$(2x + 1) = 0$

$2x + 1 - 1 = 0 - 1$

$2x = -1$

$\frac{2x}{2} = \frac{-1}{2}$

$x = -\frac{1}{2}$

Two solutions for x is not unexpected because we were solving a quadratic. To solve for y , substitute each of the solutions for x in the linear equation $y = 7x + 5$. We will also get two solutions for y .

For $x = 4$

$y = 7(4) + 5$.

$y = 28 + 5$

$y = 33$

For $x = -\frac{1}{2}$

$y = 7\left(-\frac{1}{2}\right) + 5$

$y = -\frac{7}{2} + 5$

$y = -3\frac{1}{2} + 5$

$y = 1\frac{1}{2}$

ACTIVITY 2

1. A father wants to buy a total of 5 milk drinks for his son and spend \$7.95. An eggnog costs \$1.55 and a peanut punch costs \$1.65. Calculate the number of each type of milk drink bought.

2. Solve the following simultaneous equation:

$$x + y = 14$$

$$xy = 45$$

5.9 Inequalities

This will be covered in Unit Ten.

ASSESSMENT

C X C questions

1. Solve the simultaneous equations

$$3a - 2b = 12$$

$$2a + b = 1$$

2. A restaurant bill of \$350 was paid using \$5 notes and \$50 notes. The total number of notes used was 16.

   Let x represent the number of \$5 notes.
   Let y represent the number of \$50 notes.

   (i) Write TWO equations in x and y to represent the information given.
   (ii) Hence, calculate the number of \$5 notes and the number of \$50 notes.

CONCLUSION

We looked at solving for two unknowns. The techniques acquired here will assist in solving many problems. In the lesson that follows we will look at measurement problems that involve determining the area and volume of plane shapes and figures.